Vitaly Bergelson (Ohio State University)

Mini-course "Mutually enriching connections between ergodic theory and combinatorics" (pdf)

Vitaly Bergelson (Ohio State University)
Mini-course *Mutually enriching connections between ergodic theory and combinatorics*

- The early results of Ramsey theory.

  Hilbert's irreducibility theorem, Dickson-Schur work on Fermat's equation over finite fields, van der Waerden's theorem, Ramsey's theorem and its rediscovery by Erdos and Szekeres.

- Three main principles of Ramsey theory

  First principle: Complete disorder is impossible. Second principle: Behind every 'Partition' result there is a notion of largeness which is responsible for a 'Density' enhancement of this result. Third principle: The sought-after configurations which are always to be found in large sets are abundant.

- Furstenberg's Dynamical approach.

  Partition Ramsey theory and topological dynamics Dynamical versions of van der Waerden's theorem, Hindman's theorem and Graham-Rothschild-Spencer's geometric Ramsey.

  Density Ramsey theory and Furstenberg's correspondence principle

  Furstenberg's correspondence principle. Ergodic Szemeredi's theorem. Polynomial Szemeredi theorem. Density version of the Hales-Jewett theorem.

- Stone-Cech compactifications and Hindman's theorem

  Topological algebra in Stone-Cech compactifications. Proof of Hindman's theorem via Poincare recurrence theorem for ultrafilters.

- IP sets and ergodic Ramsey theory

  Applications of IP sets and idempotent ultrafilters to ergodic-theoretical multiple recurrence and to density Ramsey theory. IP-polynomial Szemeredi theorem.

- Open problems and conjectures

If time permits:

- The nilpotent connection
- Ergodic Ramsey theory and amenable groups

Manfred Einsiedler (ETH Zürich)

Mini-course "Equidistribution on homogeneous spaces, a bridge between dynamics and number theory" (pdf)

Manfred Einsiedler (ETH Zürich)
Mini-course *Equidistribution on homogeneous spaces, a bridge between dynamics and number theory*

- Introduction to Homogeneous Dynamics

  Geodesic flow and horocycle flow on hyperbolic surfaces. The quotient SL(n,R)/SL(n,Z) and dynamics.

- Disjointness for x2 and x3 and Kloosterman sums

  We discuss briefly how disjointess results for systems related to the x2/x3 conjectures give an ineffective version of Kloosterman sums, and interpret the latter as an equidistribution result.

- Kloosterman sums and spectral gap

  We indicate how Kloosterman sums give an effective rate of mixing for the geodesic flow.

- Sparse equidistribution of primitive points

  Rational points on the periodic horocycle orbit satisfy amazing behavior under the geodesic flow. First they are on the compact period orbit, then they become equidistributed (effectively so), and finally they arrange themselves again on a periodic orbit before they venture into the cusp.

- Disjointness

  We indicate how disjointness of higher rank diagonal flows can be used to obtain further equidistribution results of rational points on periodic horocycle orbits.

Carlos Matheus Silva Santos

(IMPA Brésil)Mini-course "The Lagrange and Markov spectra from the dynamical point of view" (pdf)

Carlos Matheus Silva Santos (IMPA Brasil)
Mini-course *The Lagrange and Markov spectra from the dynamical point of view*

Lecture 1:
- Diophantine approximations, Dirichlet theorem, Hurwitz theorem, Lagrange spectrum
- Indefinite binary quadratic forms, Markov spectrum, Markov theorem, Markov tree and Vieta involutions
- Continued fraction algorithm, best rational approximations, Perron's characterization of Lagrange and Markov spectra
- Basic properties of the Lagrange and Markov spectra

Lecture 2:
- Digression: geometrical version of Perron characterization of Lagrangre spectrum (in terms of cusp excursions on the modular surface)
- Hall ray and Freiman's constant
- Moreira's theorem (and its dynamical generalizations)
- Global view on the structure of the Lagrange and Markov spectra
- Introduction to Hausdorff and box-counting dimensions

Lecture 3:
- General strategy of proof of Moreira's theorem
- Dynamical Cantor sets
- Examples of dynamical Cantor sets: affine Cantor sets and Gauss-Cantor sets
- Non-essentially affine Cantor sets
- Moreira's dimension formula
- Euler's remark
- 1st step of proof of Moreira's theorem: projections of products of Gauss-Cantor sets

Lecture 4:
- 2nd step of proof of Moreira's theorem: approximation of Lagrange and Markov spectra by projections of Gauss-Cantor sets
- 3rd step of proof of Moreira's theorem: lower semicontinuity of Hausdorff dimension across Lagrange and Markov spectra
- 4th step of proof of Moreira's theorem: upper semicontinuity of Hausdorff dimension across Lagrange and Markov spectra via an elementary compactness argument
- End of proof of Moreira's theorem: behavior of the Hausdorff dimension near 3 and near $\sqrt{12}$.

Joël Rivat (Aix-Marseille Université)

Mini-course "Introduction to Analytic Number Theory" (pdf)

Joël Rivat (Aix-Marseille University) Mini-course *Introduction to Analytic Number Theory*

By taking as a common thread some famous results and conjectures concerning prime numbers, the aim of this mini-course is to present mathematical tools which allow us to cope with them. To obtain the Prime Number Theorem, we will study arithmetic functions, Dirichlet series, properties of Riemmann ζ function and its zeros. We will show the optimal form of the large sieve using the Beurling-Selberg function and we will apply it to twin prime numbers. We will show Vinogradov's theorem (each odd number sufficiently large is the sum of three prime numbers) using the Hardy and Littlewood method. We will learn to estimate trigonometric sums using van der Corput's method which has numerous applications (complexity calculations, asymptotic formulas, discrepancy estimates). Finally, we will speak about Möbius disjointness of dynamical systems in the context of Sarnak's conjecture.

1 Prime numbers

- Tchebychev's inequality,
- Arithmetic functions,
- Dirichlet series,
- Riemann zeta function,
- Riemann's functional equation for zeta,
- The zeros of zeta,
- The Prime Number Theorem.

2 The large sieve

- The analytic forme of the large sieve,
- The Beurling-Selberg function,
- The sieve,
- The arithmetic form of the large sieve,
- Applications: twin prime numbers, sums of characters.

3 The van der Corput method

- Estimates of exponential sums,
- Applications: asymptotic formulas, discrepancy estimates.

4 Vinogradov's theorem

- The circle method,
- Major and minor arcs,
- Vaughan's identity,
- Sums of type I and II.

5 Sarnak's conjecture

- Möbius disjointness of dynamical systems - Sarnak's conjecture