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## Ergodic properties of step function skew products and the asymptotics of affine random walks.

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*The talk is based on joint work with Michael Bromberg, Nishant Chandgotia & Hitoshi Nakada.*

We consider skew products over irrational rotations of the circle $[0, 1)$ with skewing functions of form $f(x) = F([Qx])$ where $Q > 1$ is an integer. For example the well known $f(x) = 1 - 2[2x]$ as in arXiv:1603.07233.

For a "subsequence BAD" rotation of the circle, the group of essential values of such $f$ is the closed subgroup generated by the values of $F$.

For any irrational rotation, the rational ergodic properties of an ergodic skew product of this form are determined by the asymptotic "temporal statistics" values of the cocycle along the orbit of zero. These "temporal statistics" are given by an affine random walk generated by an independent flip-RAT sequence depending on $Q$, $F$ and the "minus-sign" continued fraction expansion of $Q\times$the rotation number.

In certain cases, the affine random walk satisfies a "weak, rough, local limit theorem" (as in arXiv:1603.07233) which entails rational ergodicity of the corresponding skew product.

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## On the Erdös flat polynomials problem

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In this talk I will present my recent contribution to the Erdös flat polynomials problems and its connections. I will further present some ingredients and ideas of the proof of the following fact:
*There are no square $L^2$-flat sequences of polynomials of the type*

$$\frac{1}{\sqrt{q}}(\epsilon_0 + \epsilon_1 z + \epsilon_2 z^2 + \cdots + \epsilon_{q-2} z^{q-2} + \epsilon_q z^{q-1}),$$

*where for each $j$,* $0 \leq j \leq q-1$, $\epsilon_j = \pm 1$.

As a consequence, we obtain that the Erdös-Newman conjectures on Littlewood polynomials holds. This further gives that Turyn-Golay's conjecture is true. Therefore, by appealing to Downarowicz-Lacroix result on square $L^2$-flat polynomials, we conclude that

- The special conjecture of Turyn-Golay's conjecture on the Barker sequences hold, and
- the spectrum of any continuous Morse sequences is singular. This answer one of the problem in the list of problems asked by M. Keane in his famous 1968's paper on the generalized Morse sequences.

I will further discuss some open problems related to Erdös flat polynomials.

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## On $k$-fold lebesgue functions

**Oleg Ageev** (AgeevOlegs@gmail.com)

Moscow State University, Russia

We introduce a notion being a $k$-fold lebesgue function, where any 2-fold lebesgue function is just ordinary lebesgue. We discuss how it is related to others notions in ergodic theory, mostly focusing on spectral aspects of ergodic theory.

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## Role of $r$-mixing in martingale methods for limit theorems in algebraic $N^d$-actions

**Jean-Pierre Conze** (conze@univ-rennes1.fr)

University of Rennes 1, France

For $N^d$-actions, a martingale method has been applied recently by D. Volný to ergodic sums over rectangles. For more general sets, we will discuss the needed estimates in order to use this method and their link with $r$-mixing and solutions of "$S$-unit equations" in the case of some actions by algebraic endomorphisms.

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## Furstenberg entropy values for nonsingular actions of groups without property (T)

**Alexandre I. Danilenko** (alexandre.danilenko@gmail.com)

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Let $G$ be a discrete countable infinite group that does not have Kazhdan's property (T) and let $\kappa$ be a generating probability measure on $G$. Then for each $t > 0$, there is a type $III_1$ ergodic free nonsingular $G$-action whose $\kappa$-entropy (Furstenberg entropy) is $t$.

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## Lebesgue spectrum for area preserving flows on the two torus

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Institut de Mathématiques de Jussieu – Paris Rive Gauche, France

*This is a joint work with Adam Kanigowski and Giovanni Forni.*

How much 'chaotic' can area preserving surface flows be? It is widely known (from the works of Kolmogorov, Katok and others) that starting from very low regularity these flows, if they do not have singularities, cannot be mixing. Via Poincaré sections, the latter phenomenon is due to a Denjoy type rigidity of discrete time one dimensional dynamics. However, Kochergin and then Khanin and Sinai showed that these flows can be mixing when they have singularities. Nothing however was know about their spectral type. We will explain why Kochergin flows with one (sufficiently strong) power like singularity typically have a maximal spectral type equivalent to Lebesgue measure on the circle. So, these quasi-minimal flows on the two torus, that have almost the same phase portrait as that of a minimal translation flow, share the same maximal spectral type as Anosov flows! In fact, the Lebesgue spectrum is rather reminiscent of the parabolic paradigm (of horocyclic flows for example) to which the Kochergin flows are related due to the shear along their orbits. We will discuss this relation and its consequences as well as several questions around mixing area preserving flows.

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## Approximate orthogonality of powers for ergodic affine unipotent diffeomorphisms on nilmanifolds

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Université de Lille 1, France

*Joint work with K. Frączek, J. Kułaga-Przymus and M. Lemańczyk.*

We prove that any ergodic affine unipotent diffeomorphisms of a compact nilmanifold enjoys the property of asymptotically orthogonal powers (AOP). Two consequences follow: (i) Sarnak's conjecture on Möbius orthogonality holds in every uniquely ergodic model of an ergodic affine unipotent diffeomorphism; (ii) For ergodic affine unipotent diffeomorphisms themselves, the Möbius orthogonality holds on so called typical short intervals.

## On ergodicity in periodic systems of Eaton lenses

**Krzysztof Frączek** (fraczek@mat.umk.pl)

Nicolaus Copernicus University in Toruń, Poland

*The talk is based on joint results with Martin Schmoll.*

We consider periodic Eaton lens patterns in the plane. An Eaton lens is a circular lens on the plane $\mathbb{R}^2$ which acts as a perfect retroreflector, i.e. so that each ray of light after passing through the Eaton lens is directed back toward its source. As it was proved in [1] and [2], in the simplest pattern, typically we observe highly non-ergodic behaviour. The aim of the talk is to present some exceptional families of ergodic patterns. This ergodicity is related to vanishing of Lyapunov exponents of quadratic differentials on the torus.

### References

[1] K. Frączek, M. Schmoll, *Directional localization of light rays in a periodic array of retro-reflector lenses*, Nonlinearity 27 (2014), 1689-1707.

[2] K. Frączek, R. Shi, C. Ulcigrai, *Genericity on curves and applications: pseudo-integrable billiards, Eaton lenses and gap distributions*, eprint arXiv:1508.03946.

## The structure of tame minimal dynamical systems for general groups

**Eli Glasner** (glasner@math.tau.ac.il)

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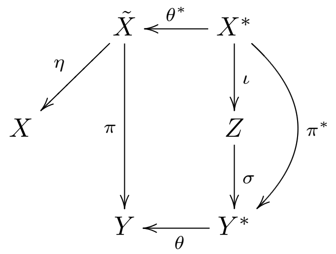
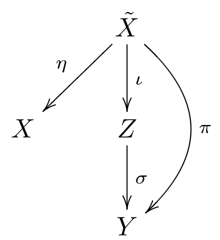
We use the structure theory of minimal dynamical systems to show that, for a general group $\Gamma$, a tame, metric, minimal dynamical system $(X, \Gamma)$ has the following structure:




Here (i) $\tilde{X}$ is a metric minimal and tame system (ii) $\eta$ is a strongly proximal extension, (iii) $Y$ is a strongly proximal system, (iv) $\pi$ is a point distal and RIM (relatively invariant measure) extension with unique section, (v) $\theta$, $\theta^*$ and $\iota$ are almost one-to-one extensions, and (vi) $\sigma$ is an isometric extension.

When the map $\pi$ is also open this diagram reduces to




In general the presence of the strongly proximal extension $\eta$ is unavoidable. However, if the system $(X, \Gamma)$ admits an invariant measure $\mu$ then $Y$ is trivial and $X = \tilde{X}$ is an almost automorphic system; i.e. $X \xrightarrow{\iota} Z$, where $\iota$ is an almost one-to-one extension and $Z$ is equicontinuous. Moreover, $\mu$ is unique and $\iota$ is a measure theoretical isomorphism $\iota : (X, \mu, \Gamma) \to (Z, \lambda, \Gamma)$, with $\lambda$ the Haar measure on $Z$. Thus, this is always the case when $\Gamma$ is amenable.

One consequence of this structure theorem is that, for an amenable group tame systems (not necessarily minimal) have discrete spectrum.

## Ordered Groups and Spectrum of Cantor Minimal Systems

**Maryam Hosseini** (maryhoseini@ipm.ir)

IPM, Tehran, Iran

*The talk is based on the joint works with Thierry Giordano and David Handelman at Uottawa and Naser Golestani at IPM.*

We will discuss the spectrum of Cantor minimal systems. To any (continuous) eigenvalue of a Cantor minimal system $(X, T)$, we associate a continuous real coboundary. So for any element in the additive group of the spectrum of a Cantor minimal system there exists a *bounded remainder* clopen set that its measure is fixed for all invariant measures in $\mathcal{M}(X)$. Using *dimesion group* homomorphisms we try to have some approaches to the converse of this proposition.

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## Topological dynamics of piecewise $\lambda$-affine maps of the interval

**Arnaldo Nogueira** (arnaldo.nogueira@univ-amu.fr)

Aix-Marseille Université, France

*The talk is based on a joint work with Benito Pires and Rafael Rosales.*

Let $-1 < \lambda < 1$ and $f : [0, 1) \to \mathbb{R}$ be a piecewise $\lambda$-affine contraction, that is, there exist points $0 = c_0 < c_1 < \cdots < c_{n-1} < c_n = 1$ and real numbers $b_1, \ldots, b_n$ such that $f(x) = \lambda x + b_i$ for every $x \in [c_{i-1}, c_i)$. We prove that, for Lebesgue almost every $\delta \in \mathbb{R}$, the map $f_\delta = f + \delta \pmod 1$ is asymptotically periodic. More precisely, $f_\delta$ has at most $n + 1$ periodic orbits and the $\omega$-limit set of every $x \in [0, 1)$ is a periodic orbit.

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## Poisson suspensions that are disjoint from Gaussian systems

**Emmanuel Roy** (roy@math.univ-paris13.fr)

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*This is a joint work with Elise Janvresse and Thierry de la Rue.*

In Ergodic Theory, Poisson Suspensions and Gaussian stationary processes form the large class of dynamical systems of "probabilistic origin". They share many properties, notably Poisson suspensions cannot be distinguished from Gaussian systems from their spectral features only. However, the converse is untrue as there exists a very special class a Gaussian system with the so-called "Foiaş-Strătilă (FS)" property whose spectrum identifies them completely. In a recent paper, we obtained the existence of a Poisson counterpart to these FS Gaussian system. The aim of this talk is to present this class of Poisson suspensions and save time to give a detailed (and essentially spectral) proof of their disjointness with all Gaussian systems.

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## Absolutely Continuous Spectrum for Parabolic Flows/Maps

**Lucia D. Simonelli** (lucia.simonelli@ictp.it)

ICTP, Italy

This talk will discuss some recent developments in the study of the spectral properties of parabolic flows and maps. More specifically, it will focus on the techniques used to determine the spectrum of the time-changes of the horocycle flow and an effort to generalize these methods to create conditions under which a general parabolic flow/map would be expected to have absolutely continuous spectrum.

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## On rank and isomorphism problem for von Neumann flows with one discontinuity

**Anton Solomko** (anthony.solomko@gmail.com)

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*Joint work with Adam Kanigowski.*

A von Neumann flow is a special flow over an irrational rotation of the circle and under a piecewise $C^1$ roof function with a non-zero sum of jumps. We consider the class of von Neumann flows with one discontinuiny. We prove that any such flow has infinite rank (does not have local rank one). We also prove that the absolute value of the slope of the roof function is a measure theoretic invariant, that is two ergodic von Neumann flows with one discontinuity are not isomorphic if the slopes of the roof functions have different absolute values, regardless of the irrational rotation in the base. As a corollary, we get an uncountable family of pairwise non-isomorphic von Neumann flows.

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## On relative spectral theory

**Jean-Paul Thouvenot** (jean-paul.thouvenot@upmc.fr)

UMPC Paris, France

A classical statement in spectral theory is that a weakly mixing isometric extension of a mixing transformation is mixing. A relative version of this fact (of which we shall describe the meaning) is true. We shall describe its proof which is essentially based on an argument of F. Parreau.

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## Degree, mixing, and absolutely continuous spectrum of cocycles with values in compact Lie groups

**Rafael Tiedra de Aldecoa** (rtiedra@mat.puc.cl)

Pontifical Catholic University, Chile

We consider skew products transformations $T_\phi$ associated to cocycles $\phi$ with values in compact Lie groups $G$.

We define the degree of $\phi$ as a suitable function on the base space, and we explain how it generalises previous definitions of degree of a cocycle. For each finite-dimensional irreducible representation $\pi$ of $G$, we define in an analogous way the degree of $\pi \circ \phi$. Under ergodicity assumptions, we show that the degree of $\phi$ reduces to a constant. As a by-product, we obtain that there is no uniquely ergodic skew product $T_\phi$ with nonzero degree if $G$ is a connected semisimple compact Lie group.

Next, we show that $T_\phi$ is mixing in the orthocomplement of the kernel of the degree of $\pi \circ \phi$, and that $T_\phi$ has purely absolutely continuous spectrum in that orthocomplement under some additional assumptions. Summing up these results for each representation $\pi$, one obtains a global result for the mixing and the absolutely continuous spectrum of $T_\phi$.

If time allows, we will present two explicit examples: when $G$ is a torus and when $G = \mathsf{U}(2)$.

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## Spectra of mixing transformations

**Sergey Tikhonov** (tikhonovc@mail.ru)

Plekhanov Russian University of Economics, Russia

We make an analog of “Ryzhikov‘s spectral machine” for mixing transformations.

More exactly, let $I$ be a finite tuple of natural numbers. The set of all nonempty products elements of $I$ is denoted by $\diamond I$. Then there exists mixing transformation $T$ such that the values set of its spectral multiplicity function is $\diamond I$.

The proof of this used the leash-metric for $\Gamma$-mixing $G$-actions [1], approximations of mixing $Z^d$-actions by $\kappa$-mixing actions [2], a generic actions theory and the Ryzhikov‘s construction of the ergodic transformations with homogeneous spectrum multiplicity $n$ [3].

### References

[1] S. V. Tikhonov, *Complete metric on mixing actions of general groups*, JDCS. 19 (1), 2013, 17–31.

[2] S. Tikhonov, *Approximation of mixing transformations*,Math Notes 95:1, 2014, 253âĂŞ266.

[3] V V Ryzhikov, *Spectral multiplicities and asymptotic operator properties of actions with invariant measure*, SB MATH. 200(12),2009, 1833–1845.

## Multiple mixing and Ratner property in mininal components of locally Hamiltonian flows

**Corinna Ulcigrai** (corinna.ulcigrai@bristol.ac.uk)

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*The talk is based on joint work with Adam Kanigowski and Joanna Kułaga-Przymus.*

Minimal components of smooth area preserving flows on surfaces of higher genus, also known as locally Hamiltonian flows, can be represented as special flows over interval exchange transformations (IETs) with asymmetric logarithmic singularities. We prove that for almost every IET in the base these flows are mixing of all orders. The proof of multiple mixing exploits quantitative shearing estimates and the "switchable" Ratner property, a variant of the Ratner property recently introduced by Fayad and Kanigowski to prove an analogous result for flows over rotations. We will explain how the underlying hyperbolic renormalization dynamics (given by the Teichmueller geodesic flow or Rauzy-Veech induction) is key to provide the full measure Diophantine conditions for IETs under which the result holds.