–ABSTRACTS–

Function Spaces and Harmonic Analysis

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Organizers: A.Borichev, S.Charpentier, H.Feichtinger, H.Youssfi, and R.Zarouf

Contents

1 Talks

1.1 Evgeny Abakumov
Localization of zeroes for Cauchy transforms

We study the 'localization of zeroes' phenomenon in spaces of Cauchy transforms of discrete measures. We also discuss its relation to other topics in analysis (weighted polynomial approximation, canonical systems of differential equations, de Branges' theory, rank-one perturbations of self-adjoint operators). Joint work with A. Baranov and Yu. Belov.

1.2 Lus Daniel Abreu
Gabor-Toeplitz and polyanalytic ensembles

We study a Determinantal Point Process constructed as follows: given a Gabor-Toeplitz (localization) operator, we select from its eigenfunctions those with eigenvalue closer to one. The reproducing kernel of the resulting finite dimensional space will be the correlation kernel of the process.

In time-frequency analysis the one-point intensity of such a process is called "the accumulated spectrogram". We derive asymptotic, non-asymptotic and weak L^2 error estimates for the accumulated spectrogram. In particular we will see that the asymptotic limit is independent of the window, reflecting a universality property typical of high-dimensional systems.

Finally we will mention a connection between time-frequency analysis and polyanalytic functions, and use it to obtain some properties of the polyanalytic Ginibre ensemble studied by Haimi and Hedenmalm.

If time allows, a brief mention on hyperbolic analogues (wavelets and Bergman spaces) will be made.

This talk includes published as well as new results from a collaboration with Karlheinz Grchenig and Jos Luis Romero at NuHAG (University of Vienna).

1.3 Nicola Arcozzi
Invariant metrics for the quaternionic Hardy space

I will discuss metrics playing for the quaternionic Hardy space a role analogous to that of the hyperbolic metric in classical Hardy space theory. Joint work with Giulia Sarfatti.

1.4 Anton Baranov
Strong M-bases of reproducing kernels and spectral theory of rank-one perturbations of selfadjoint operators

Let $\{x_n\}_{n\in N}$ be a complete and minimal system in a separable Hilbert space H, and let $\{y_n\}$ be its biorthogonal system. The system $\{x_n\}$ is said to be hereditarily complete (or a strong M-basis) if for any $x \in H$ we have $x \in \mathrm{Span}\{(x, y_n)x_n\}$. This property may be understood as a very weak form of the reconstruction of a vector x from its (formal) Fourier series $\sum_n (x, y_n)x_n$.

In 2013 we solved the spectral synthesis problem for exponential systems in $L^2(-a, a)$ (equivalently, reproducing kernels of the Paley-Wiener space PW_a^2). It turned out that the nonhereditary completeness may occur even in the case of exponential systems, though the defect of incompleteness is always at most one.

In the present talk we discuss the hereditary completeness for the reproducing kernels in Hilbert spaces of entire functions introduced by L. de Branges. One of our motivations is the relation (via a functional model) between this problem and the spectral synthesis for rank one perturbations of compact selfadjoint operators. We give a complete description of de Branges spaces where nonhereditarily complete systems of reproducing kernels exist in terms of their spectral measures. As a corollary, we obtain a series of striking examples of rank one perturbations of compact selfadjoint operators for which the spectral synthesis fails up to finite- or even infinite-dimensional defect.

The talk is based on joint works with Yurii Belov, Alexander Borichev and Dmitry Yakubovich.

1.5 Davide Barbieri
A Bargmann transform for semidirect products

We will consider the voice transform for irreducible representations of semidirect products of Lie groups obtained by the decomposition of the dual action of the quasiregular representation. This class of representations includes the Schroedinger representation as well as the scalar induced representations. They are not necessarily square-integrable, so that admissibility theory is generally developed by considering quotients of the group with respect to sufficiently large subgroups. However, the reproducing kernel Hilbert spaces obtained from the functions of positive type for the representation cannot be used on such homogeneous manifolds and the group structure is lost. We will show how, in the present setting, it is possible to characterize the norm of all such spaces in the situation of reduced square integrability. This will imply in particular that a reproducing property can be obtained in terms of the group Fourier transform for all nonzero mother wavelets. Moreover, we will see how the possibility to define a CR structure over the Lie algebra allows to characterize these spaces in terms of the previous integrability condition plus a complex regularity condition, whenever the chosen mother wavelet is a fundamental minimum of the group uncertainty principle. This construction allows to recover the known results for the classical Bargmann-Fock space, as well as to define group Bargmann-like transforms for other Lie groups such as the Euclidean Motion group. As an application we will see how the group of rigid motions of the Euclidean plane allows to reproduce the structure of activated regions in brain's visual cortex in certain experiments on visual perception (as measured with functional imaging) in terms of the associated group Bargmann transform.

1.6 Frédéric Bayart
Bohr radius and infinite dimensional holomorphy

Let $f(z) = \sum_n a_n z^n$ be an analytic function defined on the unit disk $\mathbb{D}$. A famous result of Bohr says that $\sum_n |a_n| \left(\frac{1}{3}\right)^n \leq \|f\|_\infty$ and that $\frac{1}{3}$ is optimal. We are interested in versions of this result on the polydisk. Precisely, let $n \geq 2$. The Bohr radius K_n of $\mathbb{D}^n$ is defined as the largest R such that, for any holomorphic function $\sum_\alpha a_\alpha z^\alpha$ on the polydisc $\mathbb{D}^n$, the following inequality is satisfied:

$$\sum_\alpha R^{|\alpha|} |a_\alpha| \leq \|f\|_\infty.$$

We give a precised asymptotic behaviour of K_n. Our method, which is a based on an inequality due to Bohnenblust and Hille, gives also interesting results in the context of infinite dimensional holomorphy.

1.7 Yuri Belov
How to sum a Fourier series?

A well-known description of Riesz bases from exponentials $\{e^{i\lambda t}\}$ on the interval includes two conditions, namely, a Carleson condition and a Muckenhoupt condition. In other words under these conditions the corresponding formal Fourier series $\sum (f, \tilde{e_\lambda}) e_\lambda$ converges unconditionally. It was recently proved by the A.Baranov, A.Borichev and the author that there exists a complete and minimal sequence from exponentials which satisfies the Carleson condition and there is no linear summation method for the Fourier series. We will show that there is one under the Muckenhoupt condition. The talk is based on joint work with Yu.Lyubarskii.

1.8 Roman Bessonov
Atomic decomposition of coinvariant subspaces of H^1

Coinvariant subspaces of the Hardy space H^1 have the form $K^1_\theta = H^1 \cap \bar{z}\theta\overline{H^1}$, where θ is an inner function. We construct an atomic decomposition of K^1_θ with respect to the Clark measure σ_α of the inner function θ satisfying the connected level set condition by B. Cohn. As a corollary, we prove that a Hankel operator on K^2_θ is bounded (or compact) if and only if its standard symbol is in $BMO(\sigma_\alpha)$ (correspondingly, $VMO(\sigma_\alpha)$).

1.9 Olivia Constantin
Some similar aspects of Fock spaces and Bergman spaces with rapidly decreasing weights

It is well-known that, once a weight decays fast enough, the corresponding Bergman space starts to differ considerably in some respects from standard weighted Bergman spaces. It turns out that these spaces behave in many ways similarly to Fock spaces and, therefore, techniques from one setting can be employed to gain insight into the other. We are going to explore such problems first focusing on issues like natural projections and dualities. Subsequently, we are going to study some other classes of operators acting on generalized Fock spaces. This is joint work with J. A. Pelaez.

1.10 Konstantin Dyakonov
Differentiation and factoring: Gauss-Lucas type theorems on the disk

For a fairly general function f in H^∞, we find (reasonably sharp) conditions that ensure the presence of a nontrivial inner factor for f', provided that f itself enjoys a similar property. The results that arise can be viewed as descendants of the classical Gauss–Lucas theorem on the critical points of a polynomial, as well as of its hyperbolic version (due to Walsh) dealing with finite Blaschke products.

1.11 Omar El Fallah
On the Dirichlet type spaces

Let μ be a positive Borel measure on $\mathbb{T}$, the *Dirichlet-type space* $\mathcal{D}(\mu)$ is the set of analytic functions $f \in H^2$, such that

$$\mathcal{D}_\mu(f) := \frac{1}{2\pi}\int_{\mathbb{T}}\int_{\mathbb{T}} \frac{|f(\zeta)-f(\xi)|^2}{|\zeta-\xi|^2}|d\zeta|d\mu(\xi) < \infty,$$

The space $\mathcal{D}(\mu)$ is endowed with the norm

$$\|f\|^2_\mu := \|f\|^2_{H^2} + \mathcal{D}_\mu(f).$$

In this talk we give norm estimates of the reproducing kernel of $\mathcal{D}(\mu)$. We also give a description of the invariant subspaces of the Dirichlet type spaces associated to measures with countable support. (Joint work with Youssef Elmadani and Karim Kellay).

1.12 Miroslav Englis
Analytic continuation of Toeplitz operators

Generalizing results of Rossi and Vergne for the holomorphic discrete series on symmetric domains, on the one hand, and of Chailuek and Hall for Toeplitz operators on the ball, on the other hand, we establish existence of analytic continuation of weighted Bergman spaces, in the weight (Wallach) parameter, as well as of the associated

Toeplitz operators (with sufficiently nice symbols), on any smoothly bounded strictly pseudoconvex domain. Still further extension to Sobolev spaces of holomorphic functions is likewise treated.

1.13 Konstantin Fedorovskiy
L^1-estimates of derivatives of univalent rational functions and related topics

Let $\mathbb{D}$ be the unit disk in the complex plane $\mathbb{C}$, and $\mathbb{T}$ be the unit circle. In the talk we consider the growth of the quantity

$$\ell(R) := \int_{\mathbb{T}} |R'(z)|\, dm(z)$$

for rational functions R of degree n, which are bounded and univalent in $\mathbb{D}$. It will be shown, that

$$B_b(1) \leqslant \limsup_{n\to\infty} \sup_R \frac{\log \ell(R)}{\log n} \leqslant \frac{1}{2},$$

where the supremum is taken over all rational functions R univalent in $\mathbb{D}$ and such that $\|R\|_{\infty,\mathbb{T}} \leqslant 1$, and where $B_b(t)$, $t \in \mathbb{R}$, is the integral means spectrum for bounded univalent functions. Some applications of this result to problems on regularity of boundaries of Nevanlinna domains and quadrature domains will be discussed. We also consider one related result by Dolzhenko which applies to general (non-univalent) rational functions. *The talk is based on the joint work with Anton Baranov*.

1.14 Hans G. Feichtinger
Function spaces in harmonic analysis and coorbit theory

A large variety of Banach spaces of functions or distributions, or spaces of analytic functions arising in complex analysis, are connected with the action of certain groups.

Coorbit theory, as developed jointly with Karlheinz Groechenig in the 80s allows to explore these spaces using methods from group representation theory and mostly functional analytic ideas, nowadays known is Banach frames or Riesz projection bases. Such ideas are closely related to the question of sets of interpolation and sets of sampling for suitable Hilbert or also Banach spaces of (smooth) functions, including the transform domain of e.g. the continuous wavelet transform or the STFT (short-time Fourier transform).

In this introductory presentation a perspective to the topics covered in the different presentations of this conference is given, encouraging to transfer questions and results form one setting to a related setting, where so far other questions may be dominant.

1.15 Sandrine Grellier
Multiple singular values of Hankel operators and weak turbulence

In this talk, we will show how spectral analysis of Hankel operators on the unit disc allows to understand a weak turbulence phenomenon of the solutions of some degenerate half wave equation. More precisely, we construct a nonlinear Fourier transformation on the space of symbols of compact Hankel operators which enables to obtain informations on the dynamics of the solution of the cubic Szegő equation. In particular, we exhibit a weak turbulence phenomenon for a G_δ-dense of smooth initial data.

1.16 Friedrich Haslinger
Spectral properties of the d-bar-Neumann operator

We consider the $\overline{\partial}$-Neumann operator

$$N : L^2_{(0,q)}(\Omega) \longrightarrow L^2_{(0,q)}(\Omega),$$

where $\Omega \subset \mathbb{C}^n$ is a bounded pseudoconvex domain, and

$$N_\varphi : L^2_{(0,q)}(\Omega, e^{-\varphi}) \longrightarrow L^2_{(0,q)}(\Omega, e^{-\varphi}),$$

where $\Omega \subseteq \mathbb{C}^n$ is a pseudoconvex domain and φ is a plurisubharmonic weight function. N is the inverse to the complex Laplacian $\Box = \overline{\partial}\,\overline{\partial}^* + \overline{\partial}^*\,\overline{\partial}$.

Using a general description of precompact subsets in L^2-spaces we obtain a characterization of compactness of the $\overline{\partial}$-Neumann operator, which can be applied to related questions about Schrödinger operators with magnetic field and Pauli and Dirac operators and to the complex Witten Laplacian. In this connection it is important to know whether the Fock space

$$\mathcal{A}^2(\mathbb{C}^n, e^{-\varphi}) = \{f : \mathbb{C}^n \longrightarrow \mathbb{C} \text{ entire } : \int_{\mathbb{C}^n} |f|^2 e^{-\varphi}\, d\lambda < \infty\}$$

is infinite-dimensional, which depends on the behavior at infinity of the eigenvalues of the Levi matrix of the weight function φ.

In addition we discuss obstructions to compactness of the $\overline{\partial}$-Neumann operator, and we describe, in some special cases, the spectrum of the $\Box$-operator.

1.17 Haakan Hedenmalm
Weighted integrability of polyharmonic functions and the uniqueness theorem of Holmgren

We consider L^p spaces with standard weights in the unit disk, indexed by the real parameter α. We then consider the biharmonic functions, or more generally, the N-harmonic functions. A natural question is now when the weighted integrability forces the function to vanish. We are led to consider new boundary value problems, and to see what happens for other domains as well.

1.18 Gady Kozma
Riesz bases of exponentials for finite unions of intervals

We show that for any finite union S of intervals in R, the Hilbert space $L^2(S)$ enjoys a Riesz basis all whose elements are of the form $e^{(ixt)}$. There are no arithmetic restrictions on the sizes of the intervals or their positions. Joint work with Shahaf Nitzan.

1.19 Nir Lev
Sampling on quasicrystals

Consider a function f in $L^2(R^d)$ with spectrum in a compact set S. By a result due to Matei and Meyer, f may be reconstructed in a stable way from its samples on a quasicrystal, provided that the sampling rate is greater than the critical one - namely, the Lebesgue measure of S. I will discuss some results concerning sampling at the critical rate, obtained in joint work with Gady Kozma and with Sigrid Grepstad.

1.20 Yurii Lyubarskii
Direct and inverse problem of multichannel scattering

We consider direct and inverse scattering problems for a system of particles consisting of a finite set of particles and a finite number of adjusted semi-infinite channels which are homogeneous at infinity.

1.21 Wolodymyr Madych
Convergence and summability of cardinal sine series

The class of sequences that give rise to convergent cardinal series can be completely characterized. If $\mathcal{C}$ denotes the collection of all such convergent series then $\mathcal{C} \subset E_\pi$, where E_π is the class of entire functions of exponential type no greater than π. In spite of the above mentioned characterization, it is not always a routine matter to determine whether or not a given class of functions in E_π is contained in $\mathcal{C}$. (This, of course, is usually the case with such characterizations.) In this talk we will introduce several, to our knowledge new, classes of entire functions that belong to $\mathcal{C}$. Furthermore, we confirm that several classical summability methods for cardinal sine series, including the spline summability method of Schoenberg, are regular in the sense of Hardy.

1.22 Evgenia Malinnikova
Composition operators on model spaces

We study compactness and Schatten class membership of composition operator C_ϕ acting from a model space K_θ to the Hardy space H^2. The conditions are given in terms of the interplay between ϕ and the inner function θ. The situation simplifies when the inner function satisfies the connected level set condition. The talk is based on joint works with Yu. Lyubarskii, A. Aleman and K.-M. Perfekt.

1.23 Mieczyslaw Mastylo
On interpolation of analytic families of multilinear operators

We will discuss some results concerning complex interpolation of analytic families of multilinear operators. We prove interpolation theorems for analytic families of operators defined on spaces generated by the Calderón method applied to couples of quasi-Banach lattices with nontrivial lattice convexity. As an application we obtain a multilinear version of Stein's classical interpolation theorem for analytic families of operators taking values in Lebesgue, Lorentz, and Hardy spaces. We use this theorem to prove that the bilinear Bochner-Riesz operator is bounded from $L^p(\mathbb{R}^n) \times L^p(\mathbb{R}^n)$ into $L^{p/2}(\mathbb{R}^n)$ for $1 < p < 2$. The talk is based on a joint work with L. Grafakos.

1.24 Mishko Mitkowski
Function theory in Hilbert spaces with a generalized frame

A well known method for studying the properties of Hilbert space operator(s) is by representing it(them) as simple operator(s) acting on an appropriate classical function space. In this setting we often exploit the holomorphic structure that this function space usually possesses. For this reason many of the basic operator theoretic results were developed for specific classes of operators acting on a variety of specific function spaces. In this talk, we will show that many of these fundamental notions and results, can be also developed in the context of general Hilbert spaces that possess a generalized frame (indexed by a locally compact group). These include but are not limited to: Carleson measures, sampling and interpolation, Hankel and Toeplitz operators, dilation results, etc. In the classical theory the generalized frame is often given by the family of all/some reproducing kernels, but we will present how similar results can be also obtained even for spaces that don't have any reproducing kernels.

1.25 Paul Mozolyako
On a theorem of M. Cartwright in higher dimensions

We consider harmonic functions in the unit ball of $\mathbb{R}^{n+1}$ that are unbounded near the boundary but can be estimated from above by some (rapidly increasing) radial weight w. Our main result gives some conditions on w that guarantee the estimate from below on the harmonic function by a multiple of this weight. In dimension two this reverse estimate was first obtained by M. Cartwright for the case of the power weights, $w_p(z) = (1-|z|)^{-p}$, $p > 1$, and then generalized to a wide class of regular weights by a number of authors. Joint work with E. Malinnikova and A. Logunov.

1.26 Artur Nicolau
Minimal interpolation by Blaschke products

Let $\{z_n\}$ be a sequence of points in the unit disc satisfying the Blaschke condition. Let $\{w_n\}$ be a bounded sequence of complex values. Assume $M = M(\{z_n\}, \{w_n\}) = \inf\{\|f\|_\infty : f \in H^\infty, f(z_n) = w_n, n = 1, 2, \ldots\} < \infty$. A result of A. Stray implies that for any $N > M$, there exists a Blaschke product B_N such that $NB_N(z_n) = w_n$ for any n. In the talk we will show that certain properties of the Blaschke product with zeros $\{z_n\}$ can be transferred to the Blaschke product B_N.

1.27 Morten Nielsen
Frames for decomposition spaces generated by a single function

A fairly general method to construct both isotropic and anisotropic smoothness spaces on R^d of Triebel-Lizorkin type based on structured decompositions of the frequency space will be presented. The resulting smoothness spaces have many desirable properties. For example, adapted frames for L_2 with a single compactly supported generator can be constructed, and the smoothness norm can be completely characterized by a sparseness condition on the frame coefficients.

1.28 Kryzstof Nowak
Szegö type spectral asymptotics of Gabor phase space localization operators

The talk is an overview of a series of results describing asymptotic spectral properties of localization operators defined in terms of Gabor expansions. The asymptotic parameter is the Euclidean dilation factor applied to the localization domain and it tends to infinity. The dilation factor is interpreted in terms of Planck's constant. The results on spectral asymptotics of localization operators are compared with the asymptotic expansions of semi-classical analysis, where in the limit Heisenberg boxes shrink to points, and the uncertainty vanishes. This line of research on spectral asymptotic properties of Szegö type originated out of classical works on phase space localization operators done by Ingrid Daubechies, Henry Landau and Harold Widom, and started about 50 years ago.

1.29 Alexander Olevskii
On Fourier quasicrystals

By a quasicrystal one usually means a discrete distribution of masses that has pure point spectrum. The name was inspired by the experimental discovery of quasicrystalline materials in the middle of 80-s.

It has been conjectured that if both the support and the spectrum of a measure in R^n are uniformly discrete sets then the measure has a periodic structure.

I'll present a relevant background and discuss a recent result, joint with Nir Lev, which proves the conjecture for positive measures.

1.30 Jan-Fredrik Olsen
On a sharp estimate for Hankel operators and Putnam's inequality

In this joint work with Maria Carmen Reguera, we obtain a sharp norm estimate for Hankel operators with anti-analytic symbol. This estimate improves the corresponding classical Putnam inequality for commutators of Toeplitz operators with analytic symbol by a factor of $1/2$. This answers a recent conjecture by Bell, Ferguson and Lundberg. As an application, this yields a new proof of the Saint-Venant inequality, which relates the torsional rigidity of a domain with its area. We also obtain the corresponding sharp estimates for weighted Bergman spaces.

1.31 Joaquim Ortega-Cerdá
Orthonormal flat polynomials in the sphere

This is a joint work with Jordi Marzo. In the 3-dimensional sphere, Bourgain constructed a uniformly bounded orthonormal basis of homogeneous holomorphic polynomials of degree k. In higher dimension it is still an open problem to construct such basis. Given any $\epsilon > 0$, we construct an orthonormal system of n_k uniformly bounded orthonormal polynomials of degree k in the unit sphere in R^{m+1} where $n_k \geq (1-\epsilon)dim(P_k(S^m))$. This improves some previous work of Shiffman. The new ingredients are the use of Fekete points to build up "large" Riesz sequences of reproducing kernels.

1.32 Margit Pap
The voice transforms of the Blaschke group, discretization results

Since 2005 joint with F. Schipp we have started to study the properties of the continuous voice transforms generated by representations of the Blaschke group on the Hardy space of the unit disc and the weighted Bergman spaces respectively. Analyzing the question of discretization of these voice transforms it turned out that different techniques are required. In the case of some weighted Bergman spaces the unified approach of atomic decomposition, developed by Feichtinger and Gröchenig, can be applied because the square integrability and the integrability condition are satisfied. In this way we proved that not only the characteristic function of the unit disc will generate atomic decomposition, but every function from the minimal Möbius invariant space will generate an atomic decomposition in those weighted Bergman spaces. In the case of the voice transforms generated by the representations of the Blaschke group on Bergman spaces and the Hardy space of the unit disc respectively, the unified approach of atomic decomposition cannot be applied. In the case of the Bergman space the voice transform is square integrable, but not integrable. In the case of the Hardy space even the square integrability is not satisfied. In both cases in order to discretize the voice transforms we introduced multiresolution analysis in these spaces. This constructions lead us to the construction of analytic wavelets.

1.33 Luís Pessoa
On the structure of polyharmonic Bergman spaces

I will present some new results on the structure of polyharmonic Bergman spaces over some domains in terms of the compression of the Beurling-Ahlfors transform. It will be explained how the results are a consequence of the validity of Dzhuraev's formulas, i.e. how such study can be based on the fact that the compression of the Beurling-Ahlfors transform is a power partial isometry over special domains. Theorems of Paley-Wiener type for polyharmonic Bergman spaces will be given for half-spaces. The talk is partially based on a joint work with A. M. Santos.

1.34 Götz Pfander
Symbol modulation spaces and boundedness of multilinear pseudodifferential operators

We define a class of norms on symbols of linear or multilinear pseudodifferential operators which allow to establish the boundedness of operators on modulation spaces, L^p spaces, and Sobolev spaces. We give various examples, including the bilinear and trilinear Hilbert transform.

1.35 Stefan Richter
Hankel operators and invariant subspaces of the Dirichlet shift

The Dirichlet space D is the space of all analytic functions f on the open unit disc such that f' is square integrable with respect to two-dimensional Lebesgue measure. We prove that the invariant subspaces of the Dirichlet shift are in 1-1 correspondence with the kernels of the Dirichlet-Hankel operators. We then apply this result to obtain information about the invariant subspace lattice of the space of weak products of Dirichlet functions and to some questions about approximation of invariant subspaces of D.

Most of our results hold in the context of superharmonically weighted Dirichlet spaces.

1.36 Grigori Rozenblioum
Finite rank operators in the Fock space and the $\bar{\partial}$ equation in certain spaces of distributions

The Fock space $\mathcal{F}^2$ consists of entire analytical functions in $\mathbb{C}^1$ square integrable with Gaussian weight; it is a closed subspace in L^2 with the same weight. The Toeplitz operator with symbol F, for a bounded function F, is defined as $T_F : \mathcal{F}^2 \ni u \mapsto \mathbf{P}Fu$, where $\mathbf{P}$ is the orthogonal projection from L^2 onto $\mathcal{F}^2$. The finite rank problem consists in describing such symbols F for which the operator T_F has finite rank. After description of certain previous results, we explain how to define Toeplitz operators with symbols-distributions in a certain space and then how to solve the finite rank problem for such Toeplitz operators without even assumption of their boundedness. An essential stage in the proof consists in the study of solvability and estimates for the $\bar{\partial}$ equation in proper classes of distributions.

The talk is based upon joint papers, respectively, with A.Borichev and with N.Shirokov.

1.37 Eero Saksman
Sobolev and Triebel-Lizorkin spaces via hyperbolic fillings

We consider definitions of function spaces on metric spaces via hyperbolic fillings, which leads to a natural (quasi)-conformal invariance of the definition. Talk is based on joint work with Mario Bonk (UCLA) and Tomas Soto (Helsinki)

1.38 Kristian Seip
Hankel and composition operators on spaces of Dirichlet series

I will give a survey of the operator theory that is currently evolving on Hardy spaces of Dirichlet series. We will consider recent results about multiplicative Hankel operators as introduced and studied by Helson and developments building on the Gordon–Hedenmalm theorem on bounded composition operators on the H^2 space of Dirichlet series.

1.39 Carl Sundberg
The transitive algebra problem

A transitive algebra is an algebra of bounded linear operators on a Hilbert space H containing the identity and having no nontrivial common invariant subspace. It is an old question, due to Kadison, to decide if every transitive algebra is dense in B(H) in the strong operator topology. A positive answer to this question would easily imply the hyperinvariant subspace conjecture.

We will discus what is known about this problem and present recent related work, joint with Alexandru Aleman, Karl-Mikael Perfekt, and Stefan Richter.

1.40 Pascal Thomas
Lifting maps to the spectral ball

The spectral unit ball Ω_n is the set of all matrices $M \in \mathbb{C}^{n\times n}$ with spectral radius less than 1. Let us call π the "projection" map which to a matrix M associates $\pi(M) \in \mathbb{C}^n$, the coefficients of its characteristic polynomial (essentially), in fact the elementary symmetric functions of its eigenvalues. Let $\mathbb{G}_n := \pi(\Omega_n)$.

The spectral ball appears naturally from the operator point of view, but also in questions of robust control theory. As a special case of what is called "μ-synthesis", one can consider Pick-Nevanlinna problems for maps from the disk to Ω_n. When studying those, is often useful to project the map to the symmetrized polydisk (for instance to obtain continuity results for the Lempert function, of interest to complex analysis in several variables, and related to the two-point problem): if $\psi \in \mathcal{O}(D, \Omega_n)$ and $\psi(\alpha_j) = M_j$, $1 \le j \le N$, then $\pi \circ \psi \in \mathcal{O}(D, \mathbb{G}_n)$ and $\pi \circ \psi(\alpha_j) = \pi(M_j)$, $1 \le j \le N$. Given a map $\varphi \in \mathcal{O}(D, \mathbb{G}_n)$, we are looking for necessary and sufficient conditions for this map to "lift through given matrices", i.e. find ψ as above so that $\pi \circ \psi = \varphi$. This is problematic when the matrices M_j are derogatory (i.e. do not admit a cyclic vector). There are natural necessary conditions, involving not only the values: $\varphi(\alpha_j) = \pi(M_j)$, of course, but also derivatives of φ at the points α_j. Those conditions turn out to be sufficient in small dimensions (up to 4).

1.41 Yuri Tomilov
From prime numbers to damped waves

We explain how function-theoretical methods originating from number theory can be used to study asymptotics of damped wave equations. More precisely, we present an operator-theoretical approach to deriving optimal rates of energy decay for damped wave equations on manifolds. At its core are new (quantitative) L^p-tauberian theorems for Laplace transforms.

This is joint work with C. J. K. Batty and A. Borichev.

1.42 Nikolai Vasilevski
Commutative algebras of Toeplitz operators on the unit ball

Let $\mathbb{B}^n$ be the unit ball in $\mathbb{C}^n$. Denote by $\mathcal{A}^2_\lambda(\mathbb{B}^n)$, $\lambda \in (-1, \infty)$, the standard weighted Bergman space, which is the closed subspace of $L^2_\lambda(\mathbb{B}^n)$ consisting of analytic functions. The Toeplitz operator T_a with symbol $a \in L_\infty(\mathbb{B}^n)$ acting on $\mathcal{A}^2_\lambda(\mathbb{B}^n)$ is defined as the compression of a multiplication operator on $L^2_\lambda(\mathbb{B}^n)$ onto the Bergman space, i.e., $T_a f = B_\lambda(af)$, where B_λ is the Bergman (orthogonal) projection of $L^2_\lambda(\mathbb{B}^n)$ onto $\mathcal{A}^2_\lambda(\mathbb{B}^n)$.

Note that for a generic subclass $S \subset L_\infty(\mathbb{B}^n)$ of symbols the algebra $\mathcal{T}(S)$ generated by Toeplitz operators T_a with $a \in S$ is non-commutative and practically nothing can be said on its structure. However, if $S \subset L_\infty(\mathbb{B}^n)$ has a more specific structure (e.g. induced by the geometry of $\mathbb{B}^n$, invariance under a certain group action, or with specific smoothness properties) the study of operator algebras $\mathcal{T}(S)$ is quite important and has attracted a lot of interest during the last decades.

At the turn of this century it was observed there exist many non-trivial algebras $\mathcal{T}(S)$ (both C^* and Banach) that are commutative on each standard weighted Bergman space. The talk aims to the description, classification, and the structural analysis of these commutative algebras.

The commutative C^*-algebras generated by Toeplitz operators are classified as follows: *given any maximal commutative subgroup of bihomorphisms of the unit ball $\mathbb{B}^n$, the C^*-algebra generated by Toeplitz operators whose symbols are constant on the orbits of this subgroup is commutative on each weighted Bergman space $\mathcal{A}^2_\lambda(\mathbb{B}^n)$.*

For each such commutative C^*-algebra there exists a unitary operator R, specific for each concrete algebra, such that any Toeplitz operator T_a from this algebra is *unitary equivalent to a multiplication operator*: $RT_aR^* = \gamma_a I$. This result gives us an easy access to the majority of the essential properties of Toeplitz operators, such as compactness, boundedness, spectral properties, invariant subspaces, etc.

As a leading example we analyze then the C^*-algebra generated by Toeplitz operators with radial symbols $a(z) = a(|z|)$, and show that, independently of a dimension n of the unit ball and a weight parameter $\lambda \in (-1, \infty)$, this algebra is isomorphic and isometric to the C^*-algebra of sequences $\mathrm{SO}(\mathbb{Z}_+)$ that *slowly oscillate* in the sense of Schmidt (1924):

$$\mathrm{SO}(\mathbb{Z}_+) = \Big\{ x \in \ell_\infty \colon \lim_{\frac{j+1}{k+1} \to 1} |x_j - x_k| = 0 \Big\}.$$

We present also several results on the description and classification of the commutative Banach algebras generated by Toeplitz operators. As an example we describe the Gelfand theory for an algebra in the lowest dimensional case, the two-dimensional ball $\mathbb{B}^2$.

1.43 Felix Voigtländer
Embeddings between decomposition spaces

In this talk, we will consider (Besov-type) **Fourier-side decomposition spaces** $\mathcal{D}\left(\mathcal{Q}, L^p, \ell_u^q\right)$, which are defined as the space of all distributions $u \in \mathcal{D}'\left(\mathcal{O}\right)$ for which the norm

$$\|u\|_{\mathcal{D}(\mathcal{Q},L^p,\ell_u^q)} := \left\| \left(\left\| \mathcal{F}^{-1}\left(\varphi_i \cdot u\right) \right\|_p \right)_{i\in I} \right\|_{\ell_u^q}$$

is finite. Here, $\mathcal{Q} = \left(Q_i\right)_{i\in I}$ is a covering of $\mathcal{O} \subset \mathbb{R}^d$ (considered as a subset of the frequency domain) and $\left(\varphi_i\right)_{i\in I}$ is a (suitable) partition of unity subordinate to $\mathcal{Q}$.

Under mild assumptions on the involved coverings, we will give conditions under which there is an embedding between two decomposition spaces $\mathcal{D}\left(\mathcal{Q}, L^{p_1}, \ell_u^{q_1}\right)$ and $\mathcal{D}\left(\mathcal{P}, L^{p_2}, \ell_v^{q_2}\right)$. These conditions will be formulated in terms of embeddings between discrete sequence spaces, where these sequence spaces encode the **geometric relations** between the two coverings $\mathcal{Q}, \mathcal{P}$.

Our criteria turn out to be sharp – i.e. the embedding between the decomposition spaces exists if and only if the embedding between the discrete sequence spaces exists – for certain ranges of p_1 and/or p_2 or in general under more severe assumptions on the coverings $\mathcal{Q}, \mathcal{P}$ and the weights u, v.

Finally, we discuss a number of examples, namely

1. Embeddings between different α-modulation spaces,
2. Embeddings between shearlet smoothness spaces and Besov spaces,
3. Embeddings between homogenous and inhomogenous Besov spaces,
4. Embeddings between shearlet coorbit spaces and Besov spaces.

1.44 Dragan Vukotic
The Krzyz conjecture: an extremal problem for non-vanishing bounded analytic functions

The Krzyz conjecture, posed in 1968, regards the Taylor coefficients of analytic functions in the disk which are bounded by one and do not vanish. It claims that every coefficient (corresponding to a non-constant term) should be bounded by 2/e and the only extremal functions are certain special inner functions. Even though the structure of extremal functions has been found a long time ago, the problem remains open up to this date, its non-linearity being one of the main difficulties. In this joint work with Maria J. Martin, Eric T. Sawyer, and Ignacio Uriarte-Tuero, we review the properties of extremal functions for the Krzyz problem and of some associated quantities and show that about 20 different statements regarding these quantities are equivalent to the conjecture.

1.45 Brett Wick
Bounds for localized operator with matrix Muckenhoupt weights

In this talk we will implement the scalar proof for well-localized operators to deduce related results for vector-valued functions on matrix-weighted $L^2(\mathbb{R}, \mathbb{C}^d)$. In particular, we are able to show that for a certain class of operators, they are bounded between two matrix-weighted spaces if and only if natural testing conditions hold.

Related results are also obtained when studying the boundedness of the Hilbert transform and Haar multipliers on weighted $L^2(\mathbb{R}, \mathbb{C}^d)$. In particular, we prove that:

$$\begin{aligned} \|Hf\|_{L^2(W)} &\lesssim [W]_{A_2}^{\frac{3}{2}} \log [W]_{A_2} \|f\|_{L^2(W)} \\ \|T_\sigma f\|_{L^2(W)} &\lesssim [W]_{A_2}^{\frac{3}{2}} \log [W]_{A_2} \|\sigma\|_\infty \|f\|_{L^2(W)}. \end{aligned}$$

1.46 Kehe Zhu
Products of Toeplitz operators on the Fock space

Let f and g be functions, not identically zero, in the Fock space F^2 of C^n . We show that the product $T_f T_{\bar{g}}$ of Toeplitz operators on F^2 is bounded if and only if $f = e^p$ and $g = ce^{-p}$, where c is a nonzero constant and p is a linear polynomial.