CONFERENCE

Branching Diffusions and Gaussian Free Fields in Physics, Probability and Number Theory (Part of the Jean-Morlet chair semester held by Nicola Kistler & Véronique Gayrard) - 10-14 June 2013 : Marseille (Campus St-Charles, amphi de chimie)

ABSTRACTS

Elie Aidekon, UPMC Paris: The Extremal Process in Nested Conformal Loops

By analogy with the Liouville measures constructed by Duplantier and Sheffield in the case of the Gaussian Free Field, we construct measures associated to a collection of nested conformal loops. Then, we study the extremal process associated to the maximal conformal radius. We show that it gives a decorated Poisson point process, similarly to what is already known in the branching Brownian motion case, and to what is conjectured for the GFF.

Louis-Pierre Arguin, Université de Montréal: Law and Ergodicity of the Extremal Process of Branching Brownian Motion

In this talk, I will review the recent results on the joint distribution of the particles of branching Brownian motion close to the maximum. I will describe how the ideas can be extended to prove the conjecture of Lalley and Sellke stating that the empirical (time-averaged) distribution of the maximum of branching Brownian motion converges almost surely to a Gumbel distribution (with a realization-dependent shift). These ideas can also be applied to the empirical extremal process. This is joint work with Anton Bovier and Nicola Kistler.

Julien Barral, Université Paris 13: Multifractal Analysis of Mandelbrot Measures and Related Questions

We will give an overview of recent advances in the multifractal analysis of Mandelbrot measures (critical or not), both from the geometric and statistical point of views. In particular, we will point out a quantified Erdös-Renyi law of large numbers which provides a non-trivial refinement of the standard multifractal analysis. We will also mention some KPZ formulas.

David Belius, ETH Zurich: Gumbel Fluctuations for the Cover Time of the Discrete Torus

The cover time is the first time a random walk on a finite connected graph has visited every vertex. It is also the maximum of a certain (correlated) random field, namely the field of hitting times of the vertices. This places the study of the cover time in the context of extreme values of correlated fields. One interesting special case is the cover time of a discrete torus (finite lattice with periodic boundary). In this talk I will present a result stating that, in the limit of large volume, the fluctuations of the cover time of the three and higher dimensional tori are governed by the Gumbel distribution (i.e., the maximum behaves as if the field was independent). Among the graphs where such a result is currently known, this appears to be the one where the correlations are the strongest. I will also discuss some geometric properties of the extrema for the discrete torus.

Erwin Bolthausen, University of Zurich: A New View on Lace Expansions and Self-Avoiding Random Walks

The lace expansion was introduced in a seminal paper by Brydges and Spencer to prove the diffusive behavior of (weakly) self-avoiding random walks in dimensions above 4. The expansion is quite simple, at least for the SAW, but there is considerable work needed to derive from it the diffusive behavior. We present some recent developments using contraction techniques directly in x-space. This leads to general results for solutions of convolution equations of which the one coming from the SAW is just a special case. As applications, it leads for the SAW to new results for walks in continuous space, asymmetric SAWs, and Green's functions, and for the classical situation, it leads to local results which are sharper than those obtained by other methods. This is joint work with Christine Ritzmann, Luca Avena, Felix Rubin, Remco van der Hofstad, and Gady Kozma.

Jean Philippe Bouchaud, CFM Paris: TBA

Paul Bourgade, Harvard: Random Matrices, Strong Szegö's Theorem and L-Functions
Fluctuations of random matrix theory type have been known to occur in analytic number theory since Montgomery's calculation of the pair correlation of the zeta zeros, in the microscopic regime. At the mesoscopic scale, the analogy still holds, through a limiting Gaussian field, which present an ultrametric structure similar to log-gases. In particular we will present an unconditional proof for an analogue of the strong Szegö theorem, for L-functions. This talk is based on work with Jeffrey Kuan.

Maury Bramson, U Minnesota: Convergence in Law of the Maximum of the Two-Dimensional Gaussian Free Field, Part 1: Motivation from Branching Brownian Motion
This is the first of three talks on the convergence in law of the maximum of the two-dimensional Gaussian free field; Jian Ding and Ofer Zeitouni will present the second and third talks. Here, we discuss the analysis of the same problem in the simpler settings of branching Brownian motion / branching random walk, with Gaussian increments. After summarizing the history of the problem, we proceed to discuss the crucial role played by the right tail for the convergence in law of the distribution. We then begin a discussion of the Gaussian free field problem by introducing the modified branching random walk, which we compare with the Gaussian free field.

Francis Comets, Paris-Diderot: Large Deviations for the Cover Time of Two-Dimensional Torus.
Let T n be the cover time of the two-dimensional discrete torus of linear size n by the symmetric simple random walk. For 0<a<1, we prove that the probability of the event { T_n < a E T_n } decays like a stretched exponential. The exponent entering the stretched exponential, reflects the strong dependence at criticality. One of the main methods used in the proofs is the decoupling of the walker's trace into independent excursions by means of soft local times. This is a joint work with Christophe Gallesco, Serguei Popov and Marina Vachkovskaia.

Loren Coquille, University of Geneva: Discrete GFF with Disordered Pinning on Z^d
We consider the d-dimensional discrete GFF as a generalization of 1d directed polymers. Polymers in random environment have been extensively studied over the last decade, mainly by taking advantage of the inherent renewal structure, which is not present in dimension 2 and more. Our model of random surfaces in random environment consists in putting the GFF in the presence of a square-well potential. Disorder is introduced by reward/penalty interaction coefficients, which are given by i.i.d. random variables. Under minimal assumptions on the law of the disorder, we prove that the quenched free energy associated to this model is strictly smaller than the annealed free energy whenever the latter is strictly positive. This proves that the presence of random environment affects thermodynamical properties of the model. This is a joint work with Piotr Milos (Warsaw).

Jian Ding, U of Chicago: Convergence in Law of the Maximum of the Two-Dimensional Gaussian Free Field, Part 2: Tail estimates via Sparsification and MBRW.
This is the second of three talks on the convergence in law of the maximum of the 2D GFF; Maury Bramson and Ofer Zeitouni will present the first and third talks. In this talk, we will compute the asymptotics of the right tail for the maximum, which is a crucial ingredient in the derivation of the limit law.

Bernard Derrida, ENS Paris: Simple Models of Evolution with Selection and Genealogies
It has been known for a long time that genealogies in simple models of neutral evolution are described, for a large population, by Kingman's coalescent. In presence of selection, one can show that for a whole class of models, the statistics of the genealogies follow those of the

Bolthausen-Sznitman coalescent. Therefore, the genealogies, in presence of selection, have the same tree structure as what the Parisi theory predicts for the pure states of mean field· spin glasses. This analogy between models of evolution and spin glasses breaks down when one conditions the genealogies on the speed of evolution.

Bertrand Duplantier, CEA Saclay: Schramm-Loewner Evolution and Liouville Quantum Gravity
Liouville quantum gravity in two dimensions is described by the ``random Riemannian manifold'' obtained by changing the Lebesgue measure in the plane by a random conformal factor, the exponential of the Gaussian Free Field. This ``random surface'' is believed to be the continuum scaling limit of certain discretized random surfaces (planar maps) that can be studied with combinatorics and random matrix theory.
When boundary arcs of a Liouville quantum gravity random surface are conformally welded to each other (in a boundary quantum-length-preserving way) the resulting interface is a random curve described by the Schramm-Loewner evolution (SLE). This allows one to develop a theory of quantum fractal measures, consistent with the Knizhnik-Polyakov-Zamolochikov (KPZ) relation, and to analyze their evolution under conformal welding maps related to SLE. As an application, we construct quantum length and boundary intersection measures on the SLE curve itself. (Joint work with Scott Sheffield, MIT- Math)

Nathalie Eisenbaum, Paris 6: Inequalities for Permanental Processes
Permanental processes are a natural extension of the definition of squared Gaussian processes. Each one-dimensional marginal of a permanental process is a squared Gaussian variable, but there is not always a Gaussian structure for the entire process. The interest to better know them is highly motivated by their connection with the local time process of Markov processes (Eisenbaum and Kaspi (2009)). Unfortunately, the lack of Gaussian structure for general permanental processes makes their behavior hard to handle. I will present an analogue for infinitely divisible permanental vectors, of some well-known inequalities for Gaussian vectors. This will be used to establish a necessary condition for the continuity of infinitely divisible permanental processes. A sufficient condition has been already obtained by Marcus and Rosen (2013).

Ilya Gruzberg, U of Chicago: Critical Wave Functions and their Multifractal Spectra
Single particle wave functions at Anderson transition are multifractals whose scaling properties are best described by an infinity of critical exponents, the so-called multifractal spectra. These multifractal spectra satisfy certain symmetry relations whose field-theoretical origins we uncover. We show that they follow from conformal invariance of the critical theory, which implies their general character. Furthermore, we demonstrate that for the Anderson localization problem the entire probability distribution for the local density of states possesses a symmetry arising from the invariance of correlation functions of the underlying nonlinear sigma model with respect to the Weyl group of the target space of the model. We also consider generalized multifractal spectra and their symmetries for composite operators in the sigma model. We provide a systematic and complete classification of gradientless composite operators, and establish a connection between these composite operators and the physical observables of LDOS and wave-function correlators, as well as with some transport observables.

Jon P. Keating, Bristol: Freezing and Extreme Values: from RMT to Number Theory
I will describe recent work with Yan Fyodorov in which we explore connections between freezing in the statistical mechanics of random landscape models and the extreme values taken by the characteristic polynomials of random matrices and by the Riemann zeta function on its critical line.

Antti Kupiainen, Helsinki: Critical Mandelbrot Cascades
Mandelbrot cascades are random measures on the line that have non trivial multi fractal properties. They, and related continuous cascades, have recently played prominent role in two-dimensional random geometry. They provide also simple models of spin glass phase transition which occurs as a model parameter ("temperature") is varied. I will review the basic theory of multiplicative chaos and discuss some recent results on the critical and low temperature phase.

Pierre Le Doussal, ENS Paris: Extrema of Log-correlated Fields, Freezing Transitions and Multifractality.
I review some of the methods from physics and statistical mechanics as applied to the problem of the extrema of log-correlated variables, and of the Gaussian Free Field. First, I review the Coulomb gas methods which allowed us to conjecture in 2001 the freezing transition for this class of models, with universal features such as the double log corrections and tail of distribution of free energy. We derive exact results in the high temperature phase for the log correlated variables on the circle and the interval which allow to further conjecture the exact distribution of the extremum. Finally, we apply these results to calculate the probability distribution of the counting function associated to a given level of a multifractal signal. All results are checked by numerics.

James Lee, Washingt: Cover Times, Majorizing Measures, and the Gaussian Free Field
We exhibit an intimate connection between the cover time of a finite graph and the expected maximum of its Gaussian free field (GFF). In particular, the square of the expected maximum of the GFF times the number of edges is within a universal constant factor of the cover time. This is used to positively resolve the Blanket Time conjectures of Winkler and Zuckerman (1996), as well as a question of Aldous and Fill (1994) on deterministic computation of the cover time. The proof employs the Dynkin isomorphism theory and the Fernique-Talagrand theory of majorizing measures to relate the cover time of a graph to the statistics of the extreme values of the GFF. Joint work with Jian Ding and Yuval Peres.

Oren Louidor, UCLA: The Thinned Extremal Process of the 2D Discrete Gaussian Free Field.
We consider the discrete Gaussian Free Field in a square box of side N in Z^2 with zero boundary conditions and study the joint law of its extreme values (h) and their spatial positions (x), properly centered and scaled. Restricting attention to extreme values which are also local maxima in a neighborhood of radius r_N, we show that when N, r_N tend to infinity with r_N/N tending to 0, the joint law above converges weakly to a Poisson Point Process with intensity measure Z(dx) exp{-alpha h} dh, where alpha = sqrt{2\pi} and Z(dx) is a random measure on [0,1]^2. In particular, this yields an integral representation for the law of the absolute maximum, similar to that found in the context of Branching Brownian Motion. Time permitting, I will discuss various properties of the Z measure, including connections with the derivative martingale associated with the continuum Gaussian Free Field. Joint work with Marek Biskup (UCLA).

Pascal Maillard, Weizmann Institute: Branching Brownian Motion Martingales and FKPP Travelling Waves
Branching Brownian motion, the continuous analog of branching random walk, has been known for a long time to be in a certain sense dual to the Fisher-Kolmogorov-Petrovskii-Piskouniv (FKPP) equation. This relationship has been exploited in depth, for example in the study of its extremal particles. In this talk, I will present proofs of various results regarding tail asymptotics of certain random variables related to the additive martingales of branching Brownian motion. The method of proof is the analytical study of the FKPP travelling waves combined with Tauberian theorems and the so-called transfer theorems by Flajolet and Odlyzko. Although these proofs may not transfer to the branching random walk, the aim is to show the strength of the analytical methods available in the continuous setting.

Jean-Francois Muzy, Université de Corse: Non-Stationary 1/f Noise as a Model for Log-Volatility of Financial Time Series
In this talk we briefly review in what respect random cascade models of volatility can capture the main stylized facts observed in empirical Finance. We then adress the question of the integral scale (correlation scale) which has been overlooked in the multifractal literature. This leads us to introduce a new model for volatility fluctuations that relies on a nonstationary (1/f) Gaussian process. Its properties over any finite time interval are very similar to those of standard cascade models, where the role of the integral scale is played by the sample size. Illustrations on various examples from stock indices are provided.

Miika Nikula: On Geometric Properties of Critical Lognormal Multiplicative Chaos
We consider the fine structure of Mandelbrot cascade measures and lognormal multiplicative chaos measures at the critical parameter value where the martingale constructions degenerate. We show that these measures are atomless and derive an almost sure upper bound for the mass of small sets in terms of the diameter of the set, which is shown to be optimal in the case of Mandelbrot cascades. Our analysis depends both on the renormalization needed to make the construction converge and on the exact asymptotics of the tail of the distribution of the total mass of the measure; in the case of lognormal multiplicative chaos, we extend earlier results on tail asymptotics in order to achieve our results. Finally, we find bounds for the almost sure almost everywhere fluctuations of the measures.

Dmitry Ostrovsky: Towards the Limit Lognormal Law
I will present my work on the probability distribution of the limit lognormal stochastic process as defined by Bacry et al. My talk will consist of two parts. In the first part, I will briefly review the limit lognormal construction and the connection of its moments to the Selberg integral, followed by my results on intermittency differentiation and the ensuing exact solution for intermittency expansion of the Mellin transform of the limit distribution. In the second part, I will construct a probability distribution that matches the moments and intermittency expansion of the limit lognormal distribution and survey its mathematical properties in relation to the Selberg integral, infinite divisibility and Barnes double gamma function.

Remi Rhodes, Paris Dauphine: Liouville Brownian Motion
Physicists, in particular David, Knizhnik, Polyakov et Zamolodchikov, have shown that 2d-Liouville quantum gravity is a theory of random surfaces equipped with a metric tensor, formally understood as the exponential of the Gaussian Free Field times a background metric. In this talk, I will explain how to rigorously construct the Brownian motion associated to this metric. Such a construction makes possible to rigorously define several other tools related to the metric like a heat kernel, a Green function or a Laplace-Beltrami operator. Time permitting, I will also discuss the spectral dimension 2d-Liouville quantum gravity.

Jay Rosen, CUNY: Isomorphism Theorems for Intersection Local Times of Random Interlacements
We define renormalized intersection local times for random interlacements associated with Levy processes in R^{d} and prove an isomorphism theorem relating these renormalized intersection local times with associated Wick polynomials.

Alberto Rosso, Paris Sud: Freezing Transition in Decaying Burgers Turbulence
We reveal a phase transition with decreasing viscosity in one-dimensional decaying Burgers turbulence with a power-law–correlated random profile of Gaussian-distributed initial velocities. The low-viscosity phase exhibits non-Gaussian one-point probability density of velocities. We obtain the low orders cumulants analytically. Our results, which are checked numerically, are based on combining insights in the mechanism of the freezing transition in random logarithmic potentials with an extension of duality relations discovered recently in the random matrix theory.

Nick Simm, Queen Mary and Westfield College: fBm with Hurst Index H=0 and Statistics of GUE Characteristic Polynomials
We study the behaviour of the log-mod of the characteristic polynomial log|det(x-M)| as a function of the spectral parameter x, where M is a large GUE random matrix. We reveal that for x taken inside the bulk of the spectrum, that object is intimately related to various versions of the logarithmically-correlated random Gaussian processes, in particular, to the fractional Brownian motion (fBm) with Hurst exponent H=0. As the standard definitions always assume H>0, we provide a bona fide extension of fBm to the H=0 case in terms of a certain stochastic Fourier integral.

Beatrice de Tilière, UPMC Paris: Loops in the XOR-Ising Model
The XOR-Ising model is constructed from two independent Ising models. We prove that loops separating clusters of the XOR-Ising model have the same law as loops naturally arising in a bipartite dimer model. This allows us to shed a light on a conjecture of Wilson, stating that XOR loops are level lines of the Gaussian free field. This is joint work with Cédric Boutillier.

Vincent Vargas, Dauphine: Atomic Gaussian Multiplicative Chaos and KPZ Duality
In this talk, I will present a rigorous mathematical framework to understand dual Liouville quantum gravity and the dual KPZ equation. In a first step, I will explain the construction of the dual Liouville measure; in a second step, I will explain how to formulate properly and prove the associated dual KPZ formula. Based on joint work with J. Barral, X. Jin and R. Rhodes.

Christian Webb, Helsinki: Some Tools for Proving Convergence of Multiplicative Cascade Measures
We discuss some different ways of proving that under a suitable normalization, multiplicative cascade measures converge to non-trivial random measures. The main methods we discuss are martingale arguments, recursions for generating functions and killing of the branching random walk. We also discuss some of these arguments in the context of translation invariant models.

Ofer Zeitouni, U Minnesota and Weizmann Institute: Convergence in Law of the Maximum of the Two-Dimensional Gaussian Free Field, Part 3: From Tail Estimates to Convergence of the Maximum, and Characterization of the Limit.
Convergence of the maximum of the 2D-GFF is obtained as a consequence of decomposing the field into a coarse, slow-varying Gaussian field and a family of independent copies of the GFF. A-priori estimates on the structure of near-maxima are combined with precise tail estimates on the maximum to yield both the convergence and a characterization of the maximum.

Olivier Zindy, UPMC Paris: Poisson-Dirichlet Statistics for the Extremes of Log-Correlated Gaussian Fields
Gaussian fields with logarithmically decaying correlations, such as branching Brownian motion and the 2D Gaussian free field, are conjectured to form a new universality class of extreme value statistics (notably in the work of Carpentier & Ledoussal and Fyodorov & Bouchaud). This class is the borderline case between the class of IID random variables, and models where correlations start to affect the statistics. In this talk, I will describe a general approach based on rigorous works in spin glass theory to describe features of the Gibbs measure of these Gaussian fields. I will focus on a model defined on the periodic interval [0;1]. At low temperature, we show that the normalized covariance of two points sampled from the Gibbs measure is either 0 or 1. This is used to prove that the joint distribution of the Gibbs weights converges in a suitable sense to that of a Poisson-Dirichlet variable. This is a joint work with Louis-Pierre Arguin.